Study on Spectrum Sensing Algorithms for Cognitive Radio Systems

Trung Thanh Nguyen*† and Thomas Kaiser†
*Institute of Information and Communication Technology CDiT, Posts & Telecommunications Institute of Technology, Vietnam
†Institute of Digital Signal Processing (DSV), University of Duisburg-Essen (UDE), Germany

***Abstract*—Spectrum sensing is a key ingredient in cognitive radio systems. In order to contribute the insight in the research field, this work investigates on sensing algorithm. In this paper, techniques as popular math-tools for sensing algorithms are mentioned. Due to assumption from the knowledge of primary signal, algorithms are categorized by approaches to present. Additionally, motivated by sensing in TV frequency range, the authors narrow the investigation on algorithms for the primary signals of digital video broadcasting.**

***Index Terms*—Cognitive Radio, Spectrum Sensing, Detection, TV whitespace, Simulation**

I. INTRODUCTION

According to, Federal Communications Commission [1], the spectrum is under utilized; the peak efficiency is 85% spectrum. In New York City, United States [2], spectrum occupancy is low in 30 MHz - 3.0 GHz. Therefore, the question is how to improve the utilization of spectrum. In Vietnam, utilization of spectrum in the same frequency range was measured in Ho Chi Minh City and Long An province as the work of [3]. The results showed a similar situation of spectrum utilization. There are whitespaces, also called spectral holes, in which primary signal dose not occupy.

In the context of cognitive radio, spectrum sensing aims to detect spectrum holes in frequency ranges. In TV bands, the holes are named as TV whitespaces (TVWS). At secondary users, spectrum sensing module allocates TVWS, and provides the information of the whitespace for opportunistic transmission.

Back to previous work, the concept of cognitive radio was firstly proposed by Mitola and Maguire Jr. in 1999 [4]. In US, the regulator has permitted a trial of cognitive radio network with some specific requirements in [1]. Regulators in many other countries also encourage research on spectrum sensing. Therefore, there has been a lot of works on sensing algorithm. Normally, sensing algorithms are categorized in different approaches. Each approach has certain assumptions of signal and environments.

In order to bring an insight in this research-field, this paper follows the following methodology. Firstly, this paper introduce a generic signal model which can is adopted to apply for sensing algorithms. Besides, some of main concepts and mathematic-tools are introduced. Next, sensing algorithms are grouped by the assumptions. The optimal approach is mentioned more detail as an good example to derive from the generic model to the statistic test, performance of detection, and false alarm. The other algorithms are selected from the literature review to make an overall picture of spectrum sensing. The introduced algorithms also forward a view to the consideration of sensing in TV bands.

Due to the methodology, this paper is organized as follows. The signal model is presented in Section II. Next Section III, Section IV, and Section V mention the algorithms for the optimal detection, blind detection, and feature detection, respectively. Some simulations and results are stated in Section VI. Finally, this paper is concluded by Section VII.

II. GENERIC SIGNAL MODEL

A a common signal model has a form as

$$\begin{cases} H_0 : y(n) = w(n), & n = 0, ..., N-1 \\ H_1 : y(n) = s(n) + w(n), & n = 0, ..., N-1 \end{cases} \tag{1}$$

where

$$s_i(n) = \sum_{j=1}^{P} \sum_{k=0}^{N_{ij}-1} h_{ij}\left(k\right) x\left(n-k\right) + w(n).$$

P and M are the number of primary-user transmitter and secondary-user receiver, respectively. i^{th} denotes the receiver-antenna; h_{ij} is the channel coefficient between the Tx_i of the primary user (PU) to the Rx_j of the secondary user (SU). The transmitted signal follows the complex Gaussian $x(n) \sim \mathbb{C}N\left(0, \sigma_s^2\right)$. The noise is independent and identically distributed (i.i.d.), circular symmetric complex Gaussian (CSCG) noise $w(n) \sim \mathbb{C}N\left(0, \sigma_w^2\right)$.

Two hypotheses H_0 and H_1 denote the non-existing and existing of primary signal. For binary hypotheses, H_0 and H_1 are null hypothesis and alternative hypothesis, respectively.

In a simplified model, the channel is frequency non-selective fading, and there is a single transmit antenna. The signal model given as (1) reduces to a simple one. To be convenient, the index i, which denotes for the i^{th} receive antenna, of $y_i(n)$, $x_i(n)$ and h_i, is removed. The signal model is derived as

$$\begin{cases} H_0 : y(n) = w(n), & n = 0, ..., N-1 \\ H_1 : y(n) = hx(n) + w(n), & n = 0, ..., N-1 \end{cases} \tag{2}$$

and

$$\begin{cases} H_0 : y\left(n\right) \sim \ \mathbb{C}N\left(0, R_0\right) & R_0 = \sigma_w^2 I_{MxM} \\ H_1 : y\left(n\right) \sim \ \mathbb{C}N\left(0, R_1\right) & R_1 = E\left[YY^H\right]. \end{cases} \tag{3}$$

In the context of spectrum sensing, false-alarm is equivalent to the case when the detector says "yes" with non-presence of primary signal. The probability of detection (Pd) measures the performance of detection. This probability is the ability of detector to successfully detect the presence of primary signal. Normally, Pd is maximized while keeping Pfa constant, this value of Pfa is called constant false-alarm rate (CFAR). Meanwhile, the probability of miss detection (Pms), which sums up to 1 with Pd, is counted when the detector could not detect the presence of signal. Pms measures the ineffectiveness of detector, and also the interference of opportunistic transmission to the PU.

In binary hypotheses, the likelihood ratio (LLR) of Pd and Pfa is formulated. Next, Neyman-Pearson theorem (NP) [5] is applied to maximize the Pd with a CFAR. In the case of composite hypotheses, when parameters are unknown, i.e. noise variance or signal variance, maximum likelihood estimate (MLE) is a tool to approximate the parameters. In principle, MLE are equal to mathematic-tools such as Lagrange multiplier as in [6], Karush-Kuhn-Tucker (KKT) as in [7], concave function as in [8], or taking derivation [9] to estimate the parameters. Thanks to the random matrix theory (RMT) [10], eigenvalue-based detection is proposed. The detector requires no information of noise and the structure of primary signal. The above mathematic-tools are powerful to derive algorithms which mentioned as the following sections.

III. OPTIMAL DETECTION ALGORITHM

Optimal detector is proved in [9]. The detector could be applied when SU has the perfect knowledge of PU and the environment. Under hypothesis H_0, the distribution of one sample $y\left(n\right)$ has a form as

$$p\left(y\left(n\right)|H_0,\sigma_w^2\right)=\frac{1}{\left(\pi\sigma_w^2\right)^M}\exp\left[-\frac{1}{\sigma_w^2}y^H(n)y(n)\right]. \quad (4)$$

Due to the joint distribution of N samples, Pfa is

$$p\left(Y|H_0,\sigma_w^2\right)=\prod_{n=0}^{N-1}\frac{1}{\left(\pi\sigma_w^2\right)^M}\exp\left[-\frac{1}{\sigma_w^2}y^H(n)y(n)\right]. \quad (5)$$

Similarly, under hypothesis H_1, Pd with one sample $y\left(n\right)$ is

$$p_1=p\left(y\left(n\right)|H_1,h,\sigma_s^2,\sigma_w^2\right),$$

and

$$p_1=\frac{1}{\left(\pi\right)^M\left|\sigma_s^2hh^H+\sigma_w^2I\right|}\exp\left[-y^H\left(n\right)\left(R_1\right)^{-1}y\left(n\right)\right]. \quad (6)$$

The covariance matrix of the received signal, R_1, is

$$\begin{aligned}R_1&=E\left[\left(hx\left(n\right)\right)\left(hx\left(n\right)\right)^H\right]+E\left[w\left(n\right)w^H\left(n\right)\right]\\&=E\left[\left(hx\left(n\right)\right)\left(hx\left(n\right)\right)^H\right]+\sigma_w^2I\\&=hh^HC_x+\sigma_w^2I\\&=hh^H\sigma_s^2+\sigma_w^2I.\end{aligned} \quad (7)$$

The C_x is the covariance matrix of the desired signal as

$$C_x=x\left(n\right)x^H\left(n\right)=\sigma_s^2I.$$

With the joint distribution of N samples, the probability of detection is

$$\begin{aligned}&p\left(Y|H_1,h,\sigma_s^2,\sigma_w^2\right)\\&=\prod_{n=0}^{N-1}\frac{1}{\left(\pi\right)^M\left|\sigma_s^2hh^H+\sigma_w^2I\right|}\exp\left[-y^H\left(n\right)R_1^{-1}y\left(n\right)\right]\\&=\frac{1}{\left[\left(\pi\right)^M\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)\sigma_w^{2(N-1)}\right]^N}\exp\left[-\sum_{n=0}^{N-1}\left(.\right)\right].\end{aligned} \quad (8)$$

Applying the matrix inversion Lemma R_1, which is

$$\left(A+UCV\right)^{-1}=A^{-1}-A^{-1}U\left(C^{-1}+VA^{-1}U\right)^{-1}VA^{-1}, \quad (9)$$

the inversion matrix of R_1 is

$$R_1^{-1}=\sigma_w^{-2}I-\sigma_s^2\frac{hh^H}{\frac{\sigma_w^2}{\sigma_s^2}+\|h\|^2} \quad (10)$$

The determinant of R_1 is

$$\det\left(R_1\right)=\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)\sigma_w^{2(N-1)}. \quad (11)$$

From (5) and (8), the log-likelihood function is formulated as

$$\begin{aligned}LLR&=\frac{p\left(Y|H_1,h,\sigma_s^2,\sigma_w^2\right)}{p\left(Y|H_0,\sigma_w^2\right)}\\&=\frac{\left\|h^HY\right\|^2}{\left(\frac{\sigma_w^2}{\sigma_s^2}+\|h\|^2\right)\sigma_w^2}-N\ln\left(\frac{\sigma_s^2}{\sigma_w^2}\|h\|^2+1\right).\end{aligned} \quad (12)$$

After omitting some constant components in the log-likelihood function, a statistic test is derived as

$$T_{opt}=\left\|h^HY\right\|^2\underset{H_0}{\overset{H_1}{\gtrless}}\eta_{opt}. \quad (13)$$

T_{opt} is the sum of square variables which follow complex Gaussian distribution. Therefore, T_{opt} follows Chi-squared distribution as

$$\begin{cases}H_0:\frac{T_{opt}}{\sigma_w^2\|h\|^2}=\frac{\left\|h^HY\right\|^2}{\sigma_w^2\|h\|^2}\sim\chi_{2MN}^2\\H_1:\frac{T_{opt}}{\|h\|^2\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)}\sim\chi_{2MN}^2.\end{cases} \quad (14)$$

A. The probability of false alarm - T_{opt}

Pd has a form as

$$\begin{aligned}P_{fa}&=p\left(\frac{T_{opt}\left(Y\right)}{\sigma_w^2\|h\|^2}>\frac{\eta_{opt}}{\sigma_w^2\|h\|^2}|H_0,\sigma_w^2\right)\\&=Q_{\chi_{2MN}^2}\left(\frac{\eta_{opt}}{\sigma_w^2\|h\|^2}\right)\\&=\frac{\Gamma\left(MN,\frac{\eta_{opt}}{2\sigma_w^2\|h\|^2}\right)}{\Gamma\left(MN\right)}.\end{aligned} \quad (15)$$

It notes that the freedom of distribution is $2MN$ due to the complex signal.

The $Q_{\chi^2_{2MN}}$ [5] has the form as

$$\begin{aligned} Q_{\chi^2_k}(\eta) &= \int_\eta^\infty p(x)\,dx = \int_\eta^\infty \frac{x^{\frac{k}{2}-1}\exp\left(-\frac{x}{2}\right)}{2^{k/2}\,\Gamma\left(\frac{k}{2}\right)}dx \\ &= \frac{\Gamma\left(\frac{k}{2},\frac{\eta}{2}\right)}{\Gamma\left(\frac{k}{2}\right)}, \end{aligned} \tag{16}$$

where the Gamma function $\Gamma(z) = \int_0^\infty t^{z-1}\exp(-t)\,dt$, and $\Gamma(z,\alpha) = \int_\alpha^\infty t^{z-1}\exp(-t)\,dt$ with the freedom of k.

B. The probability of detection - T_{opt}

Pd has a form as

$$\begin{aligned} P_d &= p\left(\frac{T_{opt}(Y)}{\|h\|^2\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)} > \frac{\eta_{opt}}{(.)}|H_1, h, \sigma_s^2, \sigma_w^2\right) \\ &= Q_{\chi^2_{2MN}}\left(\frac{\eta_{opt}}{\|h\|^2\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)}\right) \\ &= \frac{\Gamma\left(MN, \frac{\eta_{opt}}{2\|h\|^2\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)}\right)}{\Gamma(MN)}. \end{aligned} \tag{17}$$

IV. BLIND DETECTION ALGORITHMS

A. Energy-based detection

The energy of received signal is the sum of the squared $x(n)$. The statistic test of energy detection is

$$T_{ED} = \sum_{n=0}^{N-1}\|x(n)\|^2 > \eta_{ED}. \tag{18}$$

Recall (3), the signal model is

$$\begin{cases} H_0: y(n) \sim \mathbb{C}N(0,R_0) & R_0 = \sigma_w^2 I_{MxM} \\ H_1: y(n) \sim \mathbb{C}N(0,R_1) & R_1 = E\left[YY^H\right], \end{cases}$$

and (7) as

$$R_1 = hh^H\sigma_s^2 + \sigma_w^2 I.$$

T_{ED} follows Chi-squared distribution,

$$\begin{cases} H_0: \frac{T_{ED}}{\sigma_w^2} \sim \chi^2_{2MN} \\ H_1: \frac{T_{ED}}{\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)} \sim \chi^2_{2MN}. \end{cases} \tag{19}$$

1) The probability of false alarm - T_{ED}: Pfa of energy detection has a form as

$$\begin{aligned} P_{fa} &= p\left(\frac{T_{ED}(Y)}{\sigma_w^2} > \frac{\eta_{ED}}{\sigma_w^2}|H_0, \sigma_w^2\right) \\ &= Q_{\chi^2_{2MN}}\left(\frac{\eta_{ED}}{\sigma_w^2}\right) \\ &= \frac{\Gamma\left(MN, \frac{\eta_{ED}}{2\sigma_w^2}\right)}{\Gamma(MN)}. \end{aligned} \tag{20}$$

2) The probability of detection - T_{ED}: Pd of energy detection has a form as

$$P_d = \frac{\Gamma\left(MN, \frac{\eta_{ED}}{2\left(\sigma_s^2\|h\|^2+\sigma_w^2\right)}\right)}{\Gamma(MN)}. \tag{21}$$

B. Eigenvalue-based detection

There are some investigation on eigenvalue-based detection such as [7], [8], [11]–[15]. Eigenvalue-based detection is the methos which uses the technique of RMT. This technique is applied in principal component analysis (PCA). PCA is a useful technique to find patterns in data of high dimension. Noise and signal follow Gaussian distribution, and the covariance matrices under hypotheses H_0, H_1 are the Wishart random matrices [10]. The largest eigenvalues follow Tracy-Widom distributions (TW). Besides, the noise and signal variance can be estimated by eigenvalue decomposition (EVD) of covariance matrix. The noise variance is estimated under the both hypotheses H_0 and H_1 [8], [9], [16]. It is mathematically intractable to estimate calculate the parameter. However, A value-table of TW distribution is computed by Momar Dieng's Matlab package RMLab [17].

The following, Roy detection is mentioned as an example of this method. In the case, SU has no knowledge of signal variance σ_s^2 but noise variance σ_w^2. We calculate the eigenvalues of the covariance matrix of received signal and sort the eigenvalues in descending order ($\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_M$). Under the hypothesis H_1, the power of desired signal is estimated as the largest eigenvalue of the covariance matrix $R1$. The statistic test as [11] is

$$T_{RD} = \frac{\lambda_1}{\sigma_w^2}, \tag{22}$$

where λ_1 is the highest eigenvalue of the covariance matrix or received signal.

Under the hypothesis H_0, λ_1 follows TW distribution [10] of order 2 as

$$\begin{cases} H_0: \frac{T_{RD}-\mu_0}{\xi_0} \to W_2 \sim TW_2 \\ H_1: \frac{T_{RD}-\mu_1}{\xi_1} \to W_2 \sim TW_2, \end{cases} \tag{23}$$

with the suitable centering (μ_0, μ_1) and the suitable scaling parameter (ξ_0, ξ_1).

1) The probability of false alarm - T_{RD}:
P_{fa} as in [10] is

$$\begin{aligned} P_{fa} &= p\left(T_{RD}\left(Y\right) > \eta_{RD} | H_0, \sigma_w^2\right) \\ &= p\left(\frac{T_{RD} - \mu_0}{\xi_0} > \frac{\eta_{RD} - \mu_0}{\xi_0} | H_0, \sigma_w^2\right) \\ &= 1 - F_{TW2}\left(s\right), s = \frac{\eta_{RD} - \mu_0}{\xi_0}, \end{aligned} \tag{24}$$

where the centering (μ_0) and scaling parameters (ξ_0)

$$\mu_0 = \left(\sqrt{M-1} + \sqrt{N}\right)^2$$
$$\xi_0 = \left(\sqrt{M-1} + \sqrt{N}\right)\left(1/\sqrt{M-1} + 1/\sqrt{N}\ \right)^{1/3}.$$

2) The probability of detection - T_{RD}:
Under hypothesis H_1, with large M, N, the statistic test is asymptotically Gaussian as [11], and P_d has the form as

$$P_d \approx Q\left[\sqrt{N}\left(\frac{\eta_{RD}}{M\rho + 1} - \frac{M-1}{MN\rho} - 1\right)\right], \tag{25}$$

where $\rho = \frac{1}{\sqrt{MN}}$ and $Q\left(z\right) = \frac{1}{\sqrt{2\pi}} \int\limits_z^\infty e^{-\frac{x}{2}} dx$.

V. FEATURE DETECTION ALGORITHMS

The transmitted signal in communication systems is man-made. It has distinctive features as in [18] in comparison with nature signals. Detectors can exploit these features to improve performance and to overcome the SNR wall. The features are from modulation, coding, or formatting the scheme of signal.

A. CP detection for OFDM-based signal

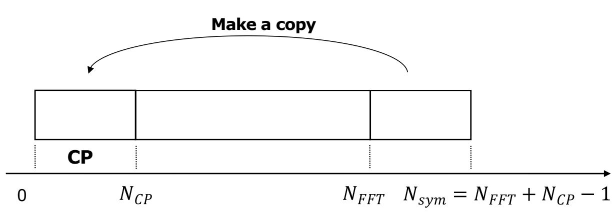

Figure 1. The cyclic prefix in OFDM-based signal.

In orthogonal frequency-division multiplexing (OFDM) transmission, it utilizes the periodicity of the cyclic prefix to detect the primary signal. The test in [19] is the maximum of correlation between samples in CP area and in the original area,

$$T_{CP} = \max_d |R_{CP}(n)| \tag{26}$$

where the correlation is defined as

$$R_{CP}(n) = \frac{1}{SN_{CP}} \sum_{u=1}^{S-1} \sum_{n=0}^{N_{CP}-1} y_{CP}\left(n\right) y_{org}\left(n\right),$$

where S is the number of segments whose length is an OFDM symbol. y_{CP} and y_{org} are the samples in CP and the original part, respectively.

B. TDSC detection for DVB-T signal

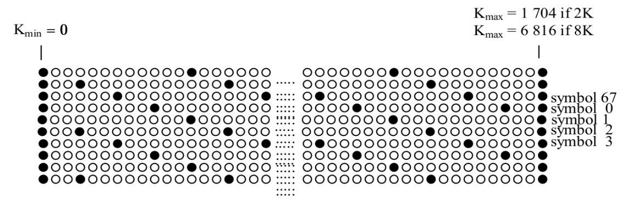

Figure 2. Fixed pilot pattern in DVB-T signal [20].

DVB-T signal [20] has a fixed pilot pattern. Based on the pattern, algorithms are proposed as in [19], [21], in which the cross-correlation of symbol in time-domain, named TDSC, is calculated as a statistic test. An advantage of the algorithm is that Pd is high with low complexity. The statistic test is

$$T_{TDSC} = \frac{1}{N} \sum_{n=0}^{N-1} y_l\left[n\right] y_m^*\left[n\right] > \eta_{TDSC}. \tag{27}$$

C. P1 detection for DVB-T2 signal

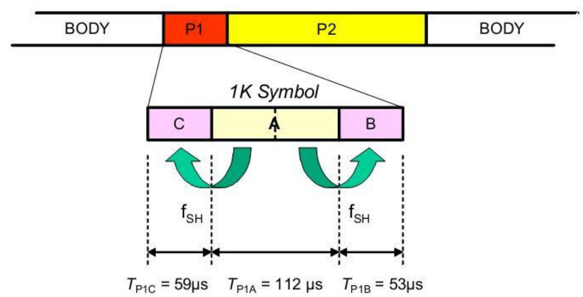

Figure 3. P1 structure in DVB-T2 signal [22].

For DVB-T2 signal [22], the algorithm which is based on the characteristic of the first preamble symbol is proposed in [23]. The observation-time of the detection is very short ($0.224ms$). This advantage helps opportunistic transmission stop quickly when the primary transmission comes back.

The statistic test is derived as

$$T_{P1} = \Re\left\{R_{N_C}(d) + R_{N_B}(d + 2N_C)\right\}, \tag{28}$$

where d denotes the index of the starting sample, and N_B and N_C are the length of the guard interval B and C as [22], respectively, and R_{N_B} and R_{N_C} are correlation from the part B and C.

VI. SIMULATIONS AND RESULTS

This section mentions simulation for some of the above algorithms. The Monte-Carlo method is used with 8000 trials for each simulation. The false alarm is kept at constant of 1%. In simulation, antenna configuration is single-input single-output.

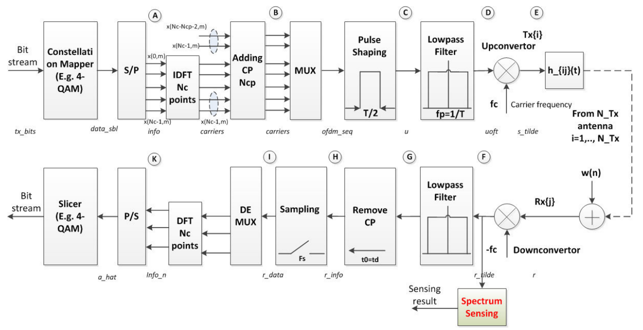

Figure 4. The position of spectrum sensing module in transmission chain.

Figure 4 shows the position of sensing module in transmission chain. In simulation, the transmitter and the sensing module are implemented. Sensing algorithms are imported in the module to validate their performance.

A. Energy detection

Figure 5 shows the performance of energy detection with/without NU. Normally, the uncertainty is from 1-2 dB [24], [25]. The performance of energy detection is sensitive with the noise factor. The inaccuracy of noise estimation degrades significantly the performance.

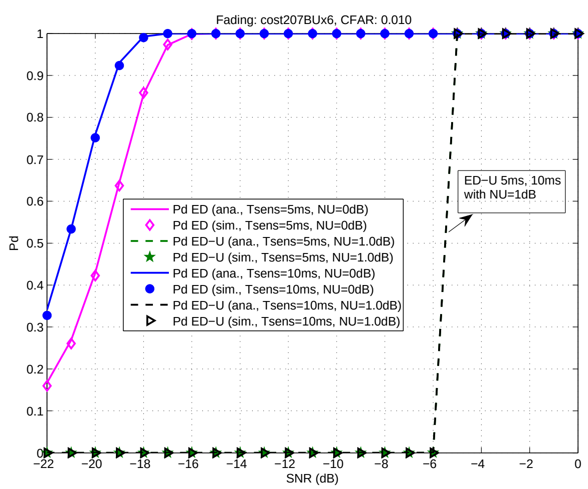

Figure 5. Energy detection with/without noise uncertainty.

As shown in Figure 5, the performance of detection is limited. The detector can not detect successfully below a certain SNR value, called SNR wall. This phenomenon is deeply investigated by research such as [24], [26]. The works prove that the reason is noise uncertainty (NU). Due to noise uncertainty, energy detectors become less reliable in low SNR. However, cognitive radio systems require detector work well in low SNR, i.e. -20 dB to avoid making interference to the primary users. This issue leads detector exploit inherent characteristics of primary signal to improve the performance.

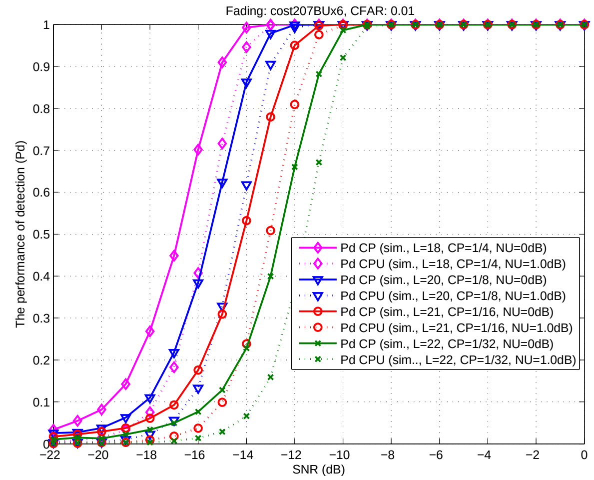

Figure 6. Cyclic prefix detection.

B. Feature detection

Figure 6 plots the performance of CP detection for OFDM-based signal. The authors used DVB-T signal [20] as an example of the primary signal. In this simulation, the sensing time is 5 ms but with different CP's ratios, and with the noise uncertainty of 0 dB and 1 dB. As shown in the figure, the performance is upgraded when increasing CP length. CP detection is verified with the signal of DVB-T standard [20] which has 4 choices for CP's ratios. The statistic test of (26) gains more accumulation when the number of sample in CP area is higher. Therefore, the performance is ordered as the configuration of $1/32$, $1/16$, $1/8$, $1/4$ correspondingly. In addition, the performance is not limited by a SNR wall as energy detection. The performance with the noise uncertainty of 1 dB is still improved when increasing the sensing time. Therefore, the requirement of high performance as low SNR such as -20 dB as in [1] could be met by CP detection with a long enough sensing-time.

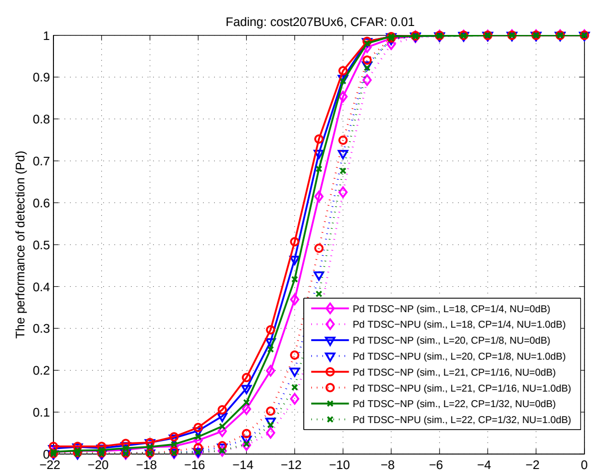

Figure 7. Time-domain symbol cross-correlation detection.

Figure 7 shows the performance of TDSC detection for DVB-T signal. This performance is not limited by a SNR wall as in energy detection. The performance with the noise uncertainty of 1 dB is degraded about 1 dB. Furthermore, TDSC detection is slightly affected by CP length.

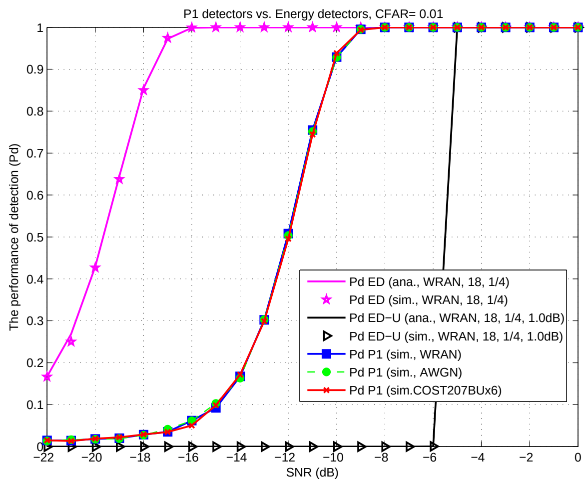

Figure 8. P1 detection.

Figure 8 depicts the performance of P1 detection for DVB-T2 signal. At the values of SNR from $-10dB$, the performance is higher than 90%. The performance does not depend on any configuration of DVB-T2 signal such as CP length, or pilot pattern. The detector works well with different types of channel. Additionally, its performance overcomes the SNR wall in energy detection.

VII. CONCLUSIONS

This paper presents some fundamental mathematic-tools for sensing algorithms. The algorithms are grouped by the knowledged of secondary user about the primary signal and the channel. The optimal detection is mentioned as examples to derive formulas of the statistic test, Pd and Pfa. Some algorithms are verified by simulation results.

REFERENCES

[1] FFC, "Federal communications commission spectrum policy task force." [Online]. Available: http://transition.fcc.gov, 2002.

[2] S. S. Company, "General survey of radio frequency bands - 30 MHz to 3 GHz." [Online]. Available: http://www.sharedspectrum.com, Sep. 2010.

[3] V. N. Q. Bao, L. Q. Cuong, L. Q. Phu, T. D. Thuan, N. T. Quy, and L. M. Trung, "Vietnam spectrum occupancy measurements and analysis for cognitive radio applications," in *Advanced Technologies for Communications (ATC), 2011 International Conference on*, pp. 135–143, 2011.

[4] I. Mitola, J. and J. Maguire, G.Q., "Cognitive radio: making software radios more personal," *Personal Communications, IEEE*, vol. 6, pp. 13 –18, Aug. 1999.

[5] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Prentice Hall, 1998.

[6] E. Axell and E. Larsson, "Comments on "multiple antenna spectrum sensing in cognitive radios"," *Wireless Communications, IEEE Transactions on*, vol. 10, pp. 1678 –1680, May 2011.

[7] R. Zhang, T. Lim, Y.-C. Liang, and Y. Zeng, "Multi-antenna based spectrum sensing for cognitive radios: A glrt approach," *Communications, IEEE Transactions on*, vol. 58, pp. 84 –88, Jan. 2010.

[8] T. J. Lim, R. Zhang, Y. C. Liang, and Y. Zeng, "Glrt-based spectrum sensing for cognitive radio," in *Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008. IEEE*, pp. 1 –5, 30 2008-Dec. 4 2008.

[9] A. Taherpour, M. Nasiri-Kenari, and S. Gazor, "Multiple antenna spectrum sensing in cognitive radios," *Wireless Communications, IEEE Transactions on*, vol. 9, pp. 814 –823, Feb. 2010.

[10] I. M. Johnstone, "On the distribution of the largest eigenvalue in principal components analysis," *The Annals of Statistics*, vol. 29, no. 2, pp. 295–327, 2001.

[11] B. Nadler, F. Penna, and R. Garello, "Performance of eigenvalue-based signal detectors with known and unknown noise level," in *Communications (ICC), 2011 IEEE International Conference on*, pp. 1 –5, Jun. 2011.

[12] M. Hamid, K. Barbe, N. Bjorsell, and W. Van Moer, "Spectrum sensing through spectrum discriminator and maximum minimum eigenvalue detector: A comparative study," in *Instrumentation and Measurement Technology Conference (I2MTC), 2012 IEEE International*, pp. 2252 –2256, May 2012.

[13] T. Swamy, S. Avasarala, T. Sandhya, and G. Ramamurthy, "Spectrum sensing: Approximations for eigenvalue ratio based detection," in *Computer Communication and Informatics (ICCCI), 2012 International Conference on*, pp. 1 –5, Jan. 2012.

[14] Y. Zeng and Y.-C. Liang, "Maximum-minimum eigenvalue detection for cognitive radio," in *Personal, Indoor and Mobile Radio Communications, 2007. PIMRC 2007. IEEE 18th International Symposium on*, pp. 1 –5, Sept. 2007.

[15] Y. Zeng and Y. chang Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *Communications, IEEE Transactions on*, vol. 57, pp. 1784 –1793, June 2009.

[16] P. Wang, J. Fang, N. Han, and H. Li, "Multiantenna-assisted spectrum sensing for cognitive radio," *Vehicular Technology, IEEE Transactions on*, vol. 59, pp. 1791 –1800, May 2010.

[17] M. Dieng, "Rmlab package." [Online]. Available: http://math.arizona.edu/ momar/research.htm, Jul. 2012.

[18] W. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," *Signal Processing Magazine, IEEE*, vol. 8, pp. 14 –36, Apr. 1991.

[19] H.-S. Chen, W. Gao, and D. Daut, "Spectrum sensing for OFDM systems employing pilot tones," *Wireless Communications, IEEE Transactions on*, vol. 8, pp. 5862 –5870, Dec. 2009.

[20] ETSI, "Digital video broadcasting (DVB); framing structure, channel coding and modulation for digital terrestrial television," European Telecommun. Standard EN300744, Jan. 2009.

[21] H.-S. Chen, W. Gao, and D. Daut, "Spectrum sensing for ofdm systems employing pilot tones and application to DVB-T OFDM," in *Communications, 2008. ICC '08. IEEE International Conference on*, pp. 3421 –3426, May 2008.

[22] ETSI, "Digital video broadcasting (DVB); frame structure channel coding and modulation for a second generation digital terrestrial television broadcasting system (DVB-T2)," European Telecommun. Standard EN302755, Apr. 2012.

[23] T. T. Nguyen, H. C. Cao, A. B. Güven, Y. Gao, K. T. Ammar, Kabbani and, and T. Kaiser, "Robust spectrum sensing of DVB-T2 signal using the first preamble symbol," in *The 2013 International Conference on Advanced Technologies for Communications, Vietnam, Hochiminh*, pp. 261 –265, Oct. 2013.

[24] R. Tandra and A. Sahai, "Fundamental limits on detection in low SNR under noise uncertainty," in *Wireless Networks, Communications and Mobile Computing, 2005 International Conference on*, vol. 1, pp. 464 – 469 vol.1, Jun. 2005.

[25] R. Tandra and A. Sahai, "SNR walls for signal detection," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 2, pp. 4 –17, Feb. 2008.

[26] D. Cabric, A. Tkachenko, and R. W. Brodersen, "Experimental study of spectrum sensing based on energy detection and network cooperation," in *The 2nd Annual International Wireless Internet Conference (WICON)*, TAPAS Workshop, Aug. 2006.